S1 · 10:10–10:25

Stability-Aware Parameter Identification for Generalized Lotka-Volterra Models

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Stability-Aware Parameter Identification for Generalized Lotka-Volterra Models

CSB Meeting 2026

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Introduction

Understanding microbial communities assembly is key for health and industrial bioprocesses

Mathematical modeling offers a powerful tool to infer interactions and evaluate community stability

Generalized Lotka-Volterra (gLV) models

$$\frac{dx_i}{dt} = x_i\left(r_i + \sum_{j=1}^{n} a_{ij}x_j - \gamma_i x_i\right), \quad i = 1, \dots, n$$

Intrinsic growth rates

Interaction parameters

Intrinsic self-regulations

$$r_i, \gamma_i > 0 \qquad a_{ij} \in \mathbb{R}$$

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Parameter estimation can be challenging due to measurement uncertainty and biological constraints

Interaction matrix

$$\begin{bmatrix} -\gamma_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\ a_{21} & -\gamma_2 & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\ a_{31} & a_{32} & -\gamma_3 & a_{34} & a_{35} & a_{36} & a_{37} \\ a_{41} & a_{42} & a_{43} & -\gamma_4 & a_{45} & a_{46} & a_{47} \\ a_{51} & a_{52} & a_{53} & a_{54} & -\gamma_5 & a_{56} & a_{57} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & -\gamma_6 & a_{67} \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & -\gamma_7 \end{bmatrix}$$

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Parameter estimation can be challenging due to measurement uncertainty and biological constraints

Stability-aware parameter estimation:
Incorporates analytical constraints and measurement noise into parameter estimation

$$\begin{bmatrix} -\gamma_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\ a_{21} & -\gamma_2 & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\ a_{31} & a_{32} & -\gamma_3 & a_{34} & a_{35} & a_{36} & a_{37} \\ a_{41} & a_{42} & a_{43} & -\gamma_4 & a_{45} & a_{46} & a_{47} \\ a_{51} & a_{52} & a_{53} & a_{54} & -\gamma_5 & a_{56} & a_{57} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & -\gamma_6 & a_{67} \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & -\gamma_7 \end{bmatrix}$$

Notation

S_I: Community whose subscript I specifies the set of species it contains

$X_{i,I}$: Equilibrium abundance of species i in the set I

$\bar{X}_{i,I}$: Measured equilibrium abundance of species i in the set I

Qualitative analysis

Normalized parameters

$$\tilde{r}_i = \frac{r_i}{\gamma_i} = \mathrm{X}_{i,i}$$

$$\tilde{a}_{ij} = \frac{a_{ij}}{\gamma_i}, i \neq j$$

Qualitative analysis

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Community properties

An equilibrium is **feasible** when all equilibrium abundances are nonnegative

$$\frac{\tilde{r}_i + \tilde{a}_{ij}\tilde{r}_j}{1 - \tilde{a}_{ij}\tilde{a}_{ji}} > 0 \qquad \frac{\tilde{r}_j + \tilde{a}_{ji}\tilde{r}_i}{1 - \tilde{a}_{ij}\tilde{a}_{ji}} > 0$$

An equilibrium is **asymptotically stable** if trajectories starting arbitrarily close stay close and converge to it over time

$$1 - \tilde{a}_{ij}\tilde{a}_{ji} > 0$$

Coexistence equilibrium

All species persist

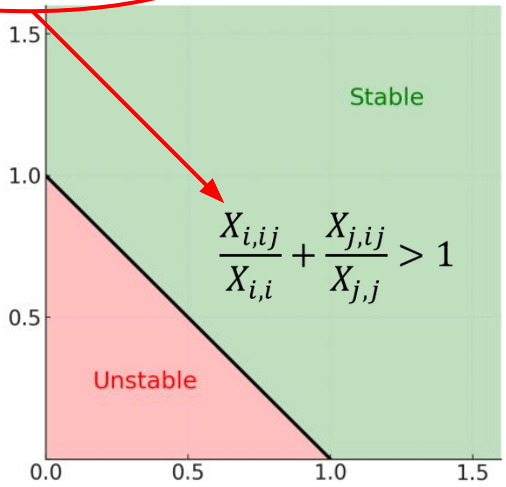
Ecologically meaningful only if it is asymptotically stable

Normalized parameters

$$\tilde{r}_i = \frac{r_i}{\gamma_i} = X_{i,i}$$

$$\tilde{a}_{ij} = \frac{a_{ij}}{\gamma_i}, i \neq j$$

$$\frac{X_{j,ij}}{X_{j,j}}$$



$$\frac{X_{i,ij}}{X_{i,i}}$$

Qualitative analysis

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General case[1,2]

Equilibrium

$$X_{1,S\{n\}} = \frac{\begin{vmatrix} X_{1,1} & -\tilde{a}_{12} & \cdots & -\tilde{a}_{1n} \\ X_{2,2} & 1 & \cdots & -\tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n,n} & -\tilde{a}_{n2} & \cdots & 1 \end{vmatrix}}{\Delta}$$

Δ is the determinant of the normalized interaction matrix

$$\vdots$$

$$X_{n,S\{n\}} = \frac{\begin{vmatrix} 1 & -\tilde{a}_{12} & \cdots & X_{1,1} \\ -\tilde{a}_{21} & 1 & \cdots & X_{2,2} \\ \vdots & \vdots & \ddots & \vdots \\ -\tilde{a}_{n1} & -\tilde{a}_{n2} & \cdots & X_{n,n} \end{vmatrix}}{\Delta}$$

Feasibility and stability conditions

$$\det \begin{bmatrix} a_1 & a_3 \\ 1 & a_2 \end{bmatrix} > 0 \qquad a_1 = \sum_{i=1}^{n} \gamma_i X_{i,S\{n\}}, \qquad a_2 = \sum_{i=1}^{n} \sum_{j<i} \gamma_i \gamma_j X_{i,S\{n\}} X_{j,S\{n\}} \begin{vmatrix} 1 & -\tilde{a}_{ij} \\ -\tilde{a}_{ji} & 1 \end{vmatrix},$$

$$\det \begin{bmatrix} a_1 & a_3 & a_5 \\ 1 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{bmatrix} > 0 \qquad a_3 = \sum_{i=1}^{n} \sum_{j<i} \sum_{k<j} \gamma_i \gamma_j \gamma_k X_{i,S\{n\}} X_{j,S\{n\}} X_{k,S\{n\}} \begin{vmatrix} 1 & -\tilde{a}_{ij} & -\tilde{a}_{ik} \\ -\tilde{a}_{ji} & 1 & -\tilde{a}_{jk} \\ -\tilde{a}_{ki} & -\tilde{a}_{kj} & 1 \end{vmatrix},$$

$$\vdots \qquad \cdots$$

$$\det \begin{bmatrix} a_1 & a_3 & a_5 & \cdots & 0 \\ 1 & a_2 & a_4 & \cdots & 0 \\ 0 & a_1 & a_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & a_{n-3} & a_{n-1} \end{bmatrix} > 0$$

For communities with more than two species, stability depends not only on interaction terms but also explicitly on the self-limitation coefficients γ_i

1. Strobeck, C. (1973). N species competition. *Ecology*, *54*(3), 650-654.
2. Barabás, G., J. Michalska-Smith, M., & Allesina, S. (2016). The effect of intra-and interspecific competition on coexistence in multispecies communities. *The American Naturalist*, *188*(1), E1-E12.

Model identification

$$Number\ of\ \text{normalized}\ parameters: n^2$$

$$Number\ of\ constraints: \sum_{k=1}^{n} k \binom{n}{k}$$

Stationary data from single species and cocultures are sufficient to determine normalized parameters

Time-series data are necessary to identify γ_i

Identification algorithm

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Stationary measurements

$$\boxed{\begin{array}{l}\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n \\ \bar{X}_{1,12}, \bar{X}_{2,12}, \ldots, \bar{X}_{n,mn}\end{array}}$$

$$\boxed{\bar{X}_I, \bar{X}_{I'}, \bar{X}_{I''}, \ldots}$$

Identification algorithm

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Stationary measurements

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Minimal approach

Apply Lognormal Noise

Identification algorithm

Stationary measurements

$\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n$

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Apply Lognormal Noise

Subsystem approach

Identification algorithm

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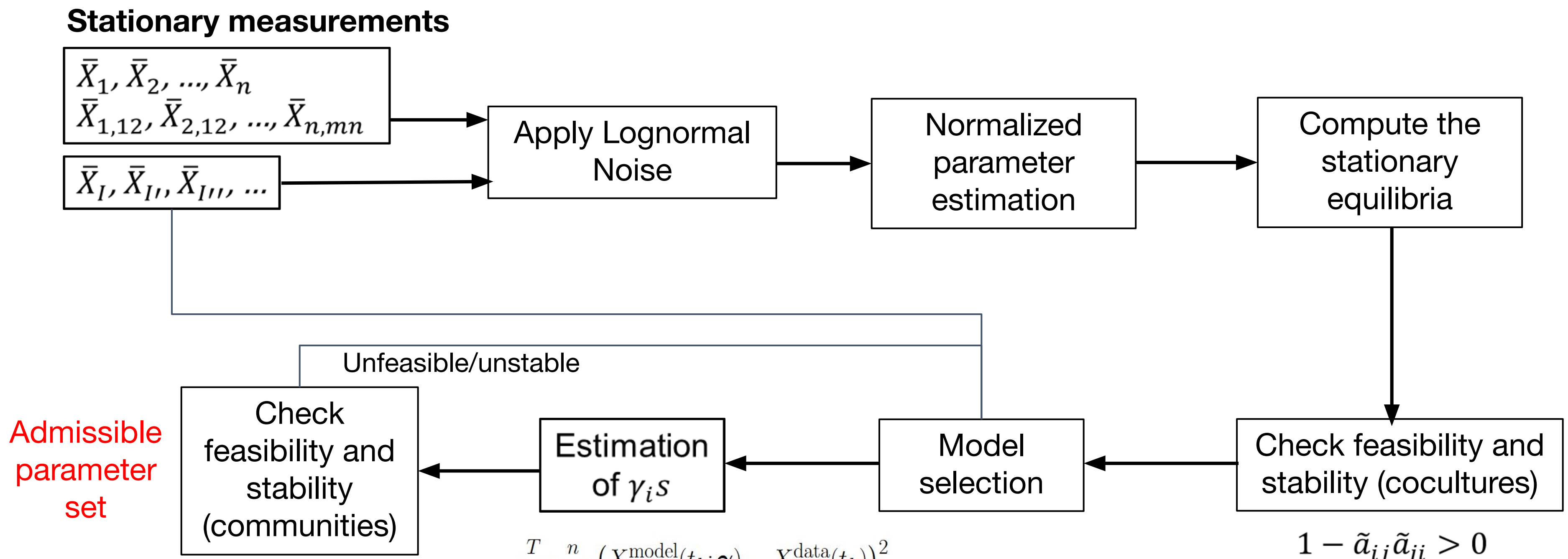


$$\mathcal{L}(\gamma) = \sum_{k=1}^{T}\sum_{i=1}^{n} \frac{\left(X_i^{\text{model}}(t_k;\gamma) - X_i^{\text{data}}(t_k)\right)^2}{\left(\mathrm{CV}_i\, X_i^{\text{data}}(t_k)\right)^2}$$

$$1 - \tilde{a}_{ij}\tilde{a}_{ji} > 0$$

$$\frac{\tilde{r}_i + \tilde{a}_{ij}\tilde{r}_j}{1 - \tilde{a}_{ij}\tilde{a}_{ji}} > 0 \quad \frac{\tilde{r}_j + \tilde{a}_{ji}\tilde{r}_i}{1 - \tilde{a}_{ij}\tilde{a}_{ji}} > 0$$

3. De Giani, A., Bovio, F., Forcella, M. E., Lasagni, M., Fusi, P., & Di Gennaro, P. (2021). Prebiotic effect of Maitake extract on a probiotic consortium and its action after microbial fermentation on colorectal cell lines. *Foods*, *10*(11), 2536.

Case study

Probiotics:

LP (X_1) : *Lactiplantibacillus plantarum*

LA (X_2): *Lactobacillus acidophilus*

BL (X_3): *Bifidobacterium animalis* subsp. *lactis*

Minimal core:

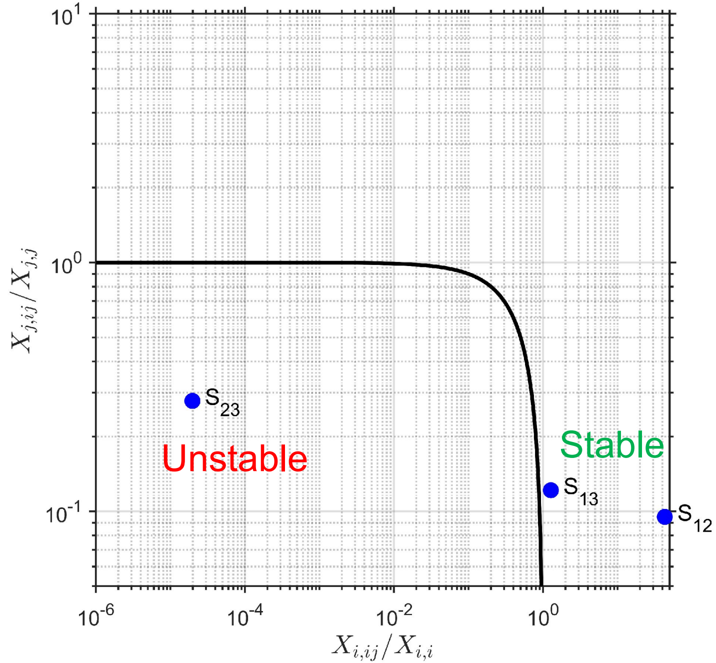
BC (X_4): *Bacteroides cellulosilyticus*

CO(X_5): *Clostridium orbiscindens*

CS (X_6): *Clostridium symbiosum*

EC (X_7): *Escherichia coli*

Maitake (2% w/v) exctract act as a prebiotic driver of community composition[3]



Pairwise coexistence diagram based on stationary abundance ratios (log–log scale)

3. De Giani, A., Bovio, F., Forcella, M. E., Lasagni, M., Fusi, P., & Di Gennaro, P. (2021). Prebiotic effect of Maitake extract on a probiotic consortium and its action after microbial fermentation on colorectal cell lines. *Foods*, *10*(11), 2536.

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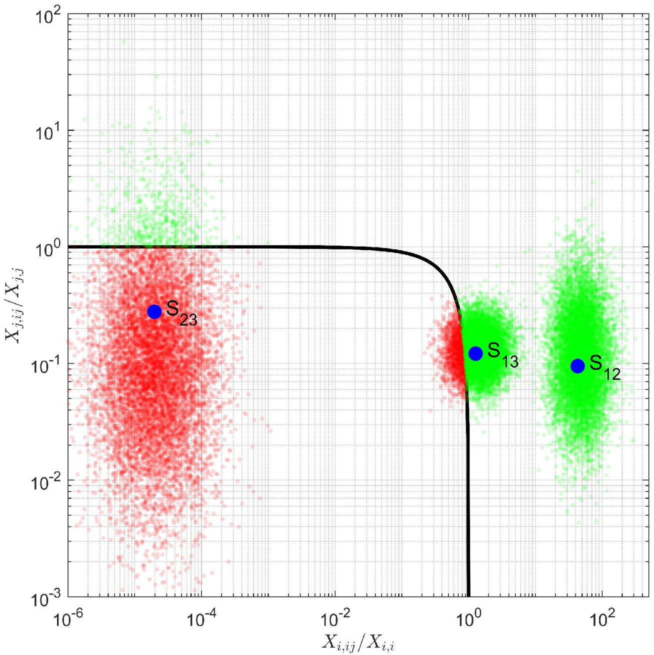

Realizations of lognormally perturbed co-cultures stationary data in the pairwise stability plane

3. De Giani, A., Bovio, F., Forcella, M. E., Lasagni, M., Fusi, P., & Di Gennaro, P. (2021). Prebiotic effect of Maitake extract on a probiotic consortium and its action after microbial fermentation on colorectal cell lines. *Foods*, *10*(11), 2536.

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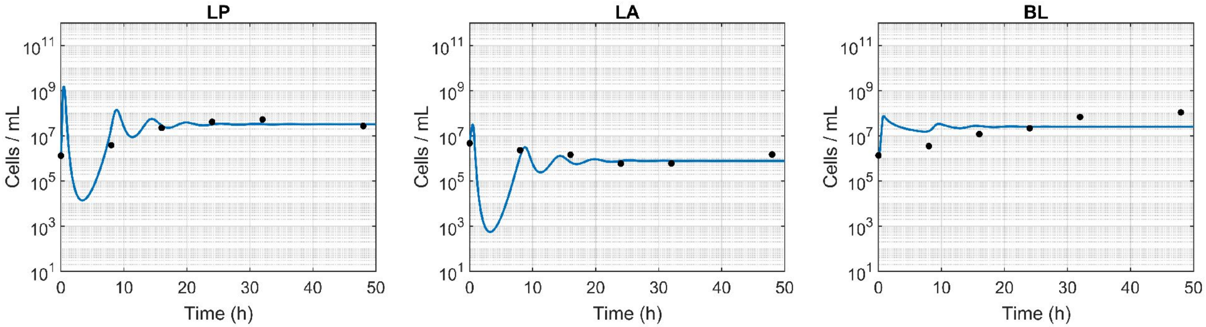

Dynamics of the probiotic consortium obtained using the minimal identification approach

3. De Giani, A., Bovio, F., Forcella, M. E., Lasagni, M., Fusi, P., & Di Gennaro, P. (2021). Prebiotic effect of Maitake extract on a probiotic consortium and its action after microbial fermentation on colorectal cell lines. *Foods*, *10*(11), 2536.

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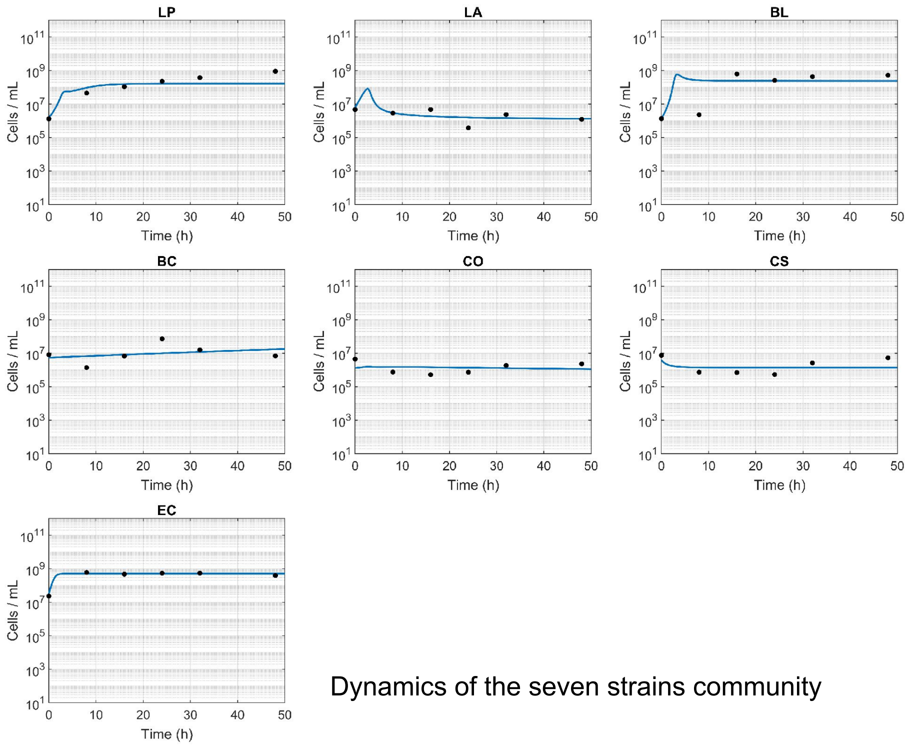

Dynamics of the seven strains community

3. De Giani, A., Bovio, F., Forcella, M. E., Lasagni, M., Fusi, P., & Di Gennaro, P. (2021). Prebiotic effect of Maitake extract on a probiotic consortium and its action after microbial fermentation on colorectal cell lines. *Foods*, *10*(11), 2536.

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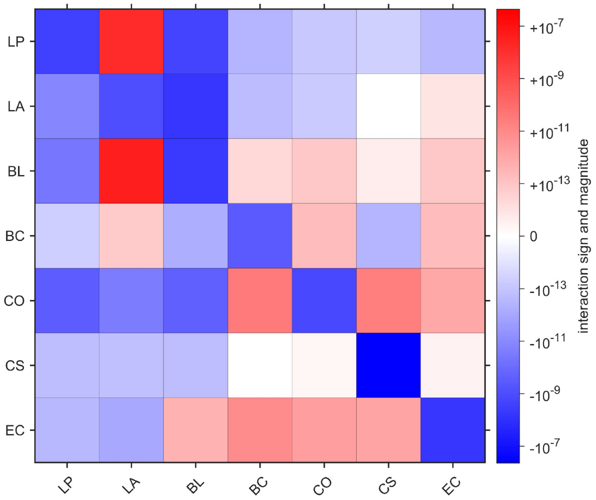
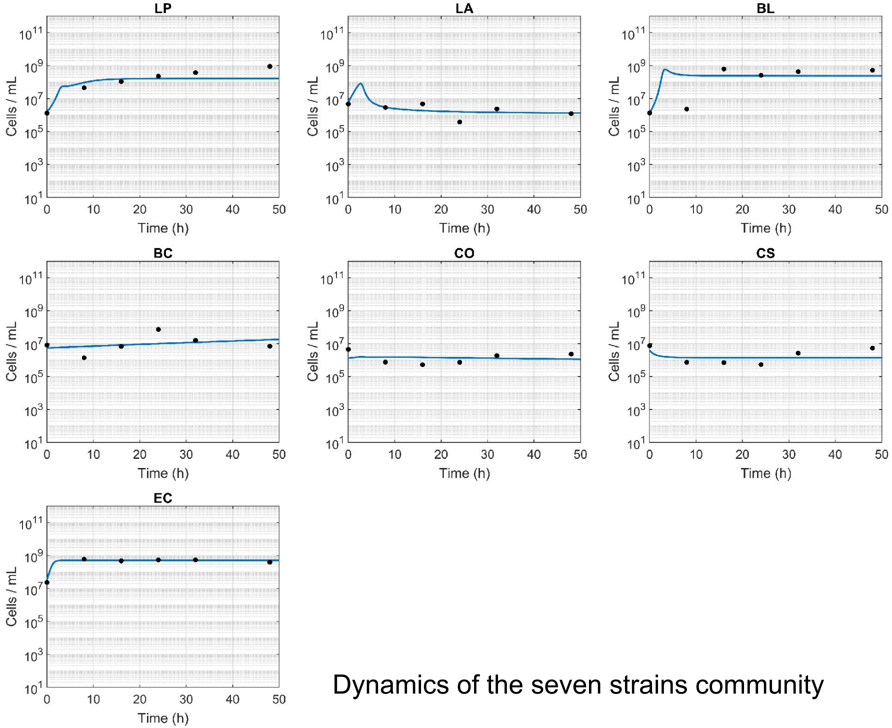


Heatmap of the interaction matrix inferred for the whole seven-strains system. The diagonal entries represent the self-limitation terms a_{ii} (-γ_i).



Dynamics of the seven strains community

4. Zenari, M., Ferraro, F., Azaele, S., Maritan, A., & Suweis, S. (2025). Generalized Lotka-Volterra systems with quenched random interactions and saturating functional response. *arXiv preprint arXiv:2506.04056.*

What’s next?

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$$\frac{dx_i}{dt} = x_i \left(r_i + \sum_{j=1}^{n} a_{ij} \frac{x_j}{M_{ij} + x_j} - \gamma_i x_i \right), \qquad i = 1, 2, \ldots, n$$

Stability condition ($\mathbf{n = 2}$)

$$\tilde{a}_{ij}\tilde{a}_{ji} \frac{M_{ij}M_{ji}}{(M_{ij}+X_{j,ij})^2 (M_{ji} + X_{i,ij})^2} < 1$$

What’s next?

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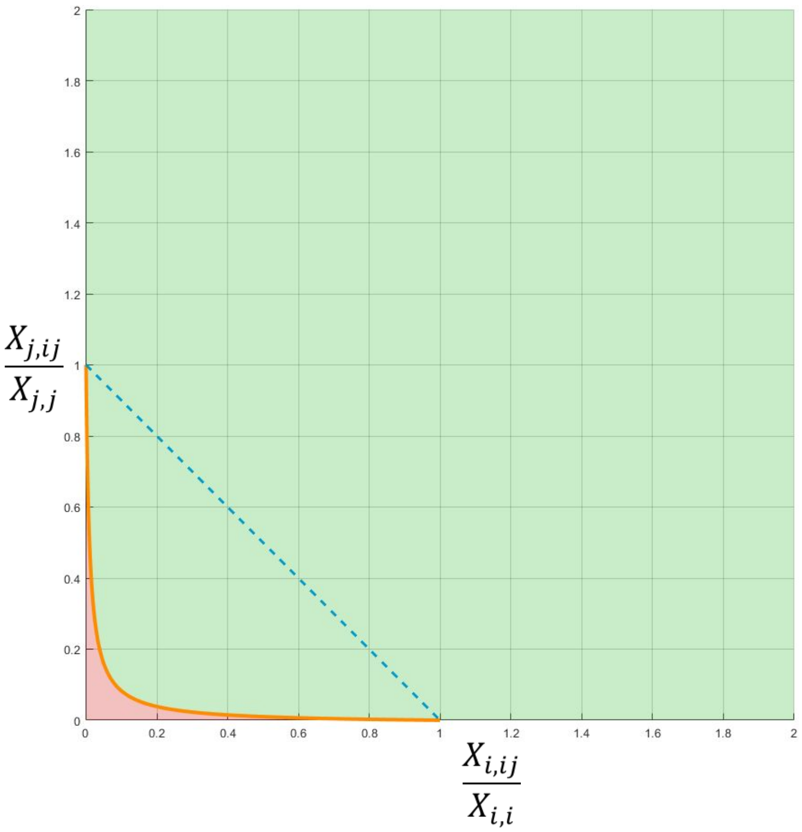
Stability condition $(\mathrm{n}=2)$

$$\tilde{a}_{ij}\tilde{a}_{ji}\frac{M_{ij}M_{ji}}{(M_{ij}+X_{j,ij})^2(M_{ji}+X_{i,ij})^2}<1$$



References

1. Strobeck, C. (1973). N species competition. *Ecology*, *54*(3), 650-654.

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Questions…